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ON INTUITIONISTIC FUZZY β GENERALIZED OPEN SETS

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ABSTRACT
In this paper, we have discussed and investigated some of the properties and some of the characterization of intuitionistic fuzzy β generalized open sets in intuitionistic fuzzy topological spaces.

INTRODUCTION
The concept of fuzzy sets was introduced by Zadeh [11] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand chang [2] introduced fuzzy topological space. In continuation of the process Coker [3] introduced intuitionistic fuzzy topological spaces. In this paper, we have discussed and investigated some of the properties and some of the characterization of intuitionistic fuzzy β generalized open sets in intuitionistic fuzzy topological spaces.

PRELIMINARIES
Definition 2.1: [1] An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,
(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [4] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy β closed set (IFβCS for short) if $int(cl(int(A))) \subseteq A$,

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(ii) intuitionistic fuzzy β open set (IFβOS for short) if A ⊆ cl(int(cl(A))).

Definition 2.5: [5**]** Let A be an IFS in an IFTS (X, τ). Then the β-interior and β-closure of A are defined as

βint(A) = ∪ {G / G is an IFβOS in X and G ⊆ A}.
βcl(A) = ∩ {K / K is an IFβCS in X and A ⊆ K}.
Note that for any IFS A in (X, τ), we have $\beta cl(A^c) = (\beta int(A))^c$ and $\beta int(A^c) = (\beta cl(A))^c$.

Result 2.6: [7] Let A be an IFS in (X, τ) then

(i) βcl(A) ⊇ A ∪ int(cl(int(A)))
(ii) βint(A) ⊆ A ∩ cl(int(cl(A)))

Definition 2.7: [7] An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy* β *generalized closed set* (IFβGCS for short) if βcl(A) ⊆ U whenever A ⊆ U and U is an IFβOS in (X, τ). The complement A^c of an IFβGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy β generalized open set (IFβGOS in short) in X.

The family of all IFβGOSs of an IFTS (X, τ) is denoted by IFβGO(X).

INTUITIONISTIC FUZZY β GENERALIZED OPEN SETS

In this section we have investigated the basic properties of an intuitionistic fuzzy β generalized open sets.

Example 3.1: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS.

Theorem 3.2: Every IFOS, IFROS [9], IFαOS [4], IFSOS [4], IFPOS [4], IFβOS [4] and IFSPOS [10] and is an IFβGOS but the converses are not true in general.
Proof: Straightforward.

Example 3.3: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then *τ* = {0~, G, 1~} is an IFT on X. Let A = ⟨ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFOS in X, since int(A) = G ≠ A.

Example 3.4: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFROS in X, since int(cl(A)) = int(1~) = 1~ ≠ A.

Example 3.5: Let X = {a, b} and G = ⟨x, $(0.5_a, 0.4_b)$, $(0.5_a, 0.6_b)$⟩. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$⟩ be an IFS in X.

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Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

We have $A^c \subseteq G$. As βcl(A^c) = A^c, βcl(A^c) ⊆ G, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFαOS in X, since int(cl(int(A))) = int(cl(G)) = int(G^c) = G ⊉ A.

Example 3.6: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨ x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

We have $A^c \subseteq G$. As βcl(A^c) = A^c, βcl(A^c) ⊆ G, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS. But it is not an IFSOS in X, since cl(int(A)) = cl(G) = G^c ⊉ A.

Example 3.7: Let X = {a, b} and G = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \ge 0.5$, $\mu_a < 0.5$ whenever $\mu_b \ge 0.6$, $0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

Now $A^c \subseteq$ 1 ∼. As βcl(A^c) = 1 ∼ ⊆ 1 ∼. We have A^c is an IFβGCS in X and hence A is an IFβGOS in X. But it is not an IFPOS, since A ⊈ int(cl(A)) = int(G^c) = 0 ∼

Example 3.8: Let X = {a, b} and G = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩, then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.5_a, 0.2_b), (0.5_a, 0.8_b)⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / provided $\mu_b < 0.7$ whenever $\mu_a \ge 0.5$, $\mu_a < 0.5$ whenever $\mu_b \ge 0.7$, $0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

Now $A^c \subseteq$ 1 ∼ and βcl(A^c) = 1 ∼ ⊆ 1 ∼. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS in X. But it is not an IFβOS, since cl(int(cl(A))) = cl(int(G^c)) = cl(0 ∼) = 0 ∼ ⊉ A.

Example 3.9: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0∼, G, 1∼} is an IFT on X. Let A = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩ be an IFS in X.

Then, IFβC(X) = {0∼, 1∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

Here A^c is an IFβCS in X, as int(cl(int(A^c))) = 0 ∼ ⊆ A. Therefore A^c is an IFβGCS in X and hence A is an IFβGOS.

Since IFPC(X) = {0 ∼, 1 ∼, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_b \ge 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \ge 0.5$, $0 \le \mu_a+\nu_a \le 1$ and $0 \le \mu_b+\nu_b \le 1$}.

But A is not an IFSPOS in X, since there exists no IFPOS B such that A ⊆ B ⊆ cl(A) in X.

Remark 3.10: Every IFGOS and every IFβGOS are independent to each other.

Example 3.11: Let X = {a, b} and G_1 = ⟨x, (0.5_a, 0.5_b), (0.5_a, 0.5_b) ⟩ and G_2 = ⟨x, (0.3_a, 0.1_b), (0.7_a, 0.8_b)⟩. Then τ = {0∼, G_1, G_2, 1∼} is an IFT on X. Let A = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩ be an IFS in X. Then $A^c \subseteq G_1$ and cl(A^c) = $G_1^c \subseteq G_1$.

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Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $\mu_a \geq 0.5$ and $\mu_b \geq 0.5$ or $\mu_a < 0.3$ and $\mu_b < 0.1$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

Therefore A^c is an IFGCS in X but it is not an IFβGCS in X and hence A is an IFGOS in X. But it is not an IFβGOS, since βint(A) ⊉ U whenever A ⊇ U and U is an IFβCS in (X, τ).

Example 3.12: Let X = {a, b} and G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩ be an IFS in X.

Then, IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ /$0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

We have $A^c \subseteq G$. As $\beta cl(A^c) = A^c$, $\beta cl(A^c) \subseteq G$, where G is an IFβOS in X. This implies that A^c is an IFβGCS in X and hence A is an IFβGOS in X. But it is not an IFGOS in X, since $G^c \nsubseteq int(A)$ whenever $G^c \subseteq A$, where G^c is an IFβCS in X.

Remark 3.13: The intersection of any two IFβGOS is not an IFβGOS in general as seen from the following example.

Example 3.14: Let X = {a, b} and τ = {0 ~, G_1, G_2, 1 ~} where G_1 = ⟨x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)⟩ and G_2 = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩. Then the IFSs A = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ and B = ⟨x, (0.4_a, 0.2_b), (0.4_a, 0.8_b)⟩ are IFβGOSs in (X, τ) but A ∩ B is not an IFβGOS in (X, τ). Let us prove A^c and B^c are IFβGCS.

Then IFβC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / provided $\mu_b < 0.7$ whenever $\mu_a \geq 0.6$, $\mu_a < 0.6$ whenever $\mu_b \geq 0.7$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

As $\beta cl(A^c) = A^c$, A^c is an IFβGCS in X and hence A is an IFβGOS in X, and $\beta cl(B^c) = B^c$, we have B^c is an IFβGCS in X and hence B is an IFβGOS in X.

Now to prove A ∩ B = ⟨x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)⟩ is an IFβGOS. Let us prove $(A \cap B)^c$ is an IFβGCS. Now since $(A \cap B)^c$ = ⟨x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)⟩ ⊆ G_1 but $\beta cl\,(A \cap B)^c = 1\sim \nsubseteq G_1$.

Therefore $(A \cap B)^c$ is not an IFβGCS in X and hence A ∩ B is not an IFβGOS in X.

Theorem 3.15: Let (X, τ) be an IFTS. Then for every A ∈ IFβGO(X) and for every B ∈ IFS(X), βint(A) ⊆ B ⊆ A ⇒ B ∈ IFβGO(X).

Proof: Let A be any IFβGOS of X and B be any IFS of X. Let βint(A) ⊆ B ⊆ A. Then A^c is an IFβGCS and $A^c \subseteq B^c \subseteq \beta cl(A^c)$. Therefore B^c is an IFβGCS [7] which implies B is an IFβGOS in X. Hence B ∈ IFβGO(X).

Theorem 3.16: An IFS A of an IFTS (X, τ) is an IFβGOS if and only if F ⊆ βint(A) whenever F is an IFβCS and F ⊆ A.

Proof: Necessity: Suppose A is an IFβGOS. Let F be an IFβCS such that F ⊆ A. Then F^c is an IFβOS and $A^c \subseteq F^c$. By hypothesis A^c is an IFβGCS, we have $\beta cl(A^c) \subseteq F^c$. Therefore F ⊆ βint(A).

Sufficiency: Let F be an IFβCS such that F ⊆ A and F ⊆ βint(A). Then $(\beta int(A))^c \subseteq F^c$ and $A^c \subseteq F^c$. This implies that $\beta cl(A^c) \subseteq F^c$, where F^c is an IFβOS. Therefore A^c is an IFβGCS. Hence A is an IFβGOS.

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Theorem 3.17: Let (X, τ) be an IFTS. Then for every A ∈ IFS(X) and for every B ∈ IFβO(X), B ⊆ A ⊆ int(cl(int(B))) ⇒ A ∈ IFβGO(X).

Proof: Let B be an IFβOS. Then B ⊆ cl(int(cl(B))). By hypothesis, A ⊆ int(cl(int(B))) ⊆ int(cl(int(cl(int(cl(B)))))) ⊆ int(cl(cl(int(cl(B))))) = int(cl(int(cl(B)))) ⊆ int(cl(cl(A))) ⊆ int(cl(A)) as B ⊆ A. Therefore A is an IFPOS and by Theorem 3.2, A is an IFβGOS. Hence A ∈ IFβGO(X).

Theorem 3.18: If A is an IFRCS and B is an IFβOS, then A ∪ B is an IFβGOS.

Proof: Let B be an IFβOS and A be an IFRCS. Then B ⊆ cl(int(cl(B))) and cl(int(A)) = A. Therefore A ∪ B ⊆ A ∪ (cl(int(cl(B))) = cl(int(A)) ∪ cl(int(cl(B))) ⊆ cl(int(cl(A))) ∪ cl(int(cl(B))) = cl(int(cl(A)) ∪ int(cl(B))) ⊆ cl(int(cl(A) ∪ cl(B))) ⊆ cl(int(cl(A ∪ B))). Therefore A ∪ B is an IFβOS and by Theorem 3.2, A ∪ B is an IFβGOS.

Theorem 3.19: Let (X, τ) be an IFTS then for every A ∈ IFSPO(X) and for every IFS B in X, A ⊆ B ⊆ cl(A) ⇒ B ∈ IFβGO(X).

Proof: Let A be an IFSPOS in X. Then there exists an IFPOS, (say) C such that C ⊆ A ⊆ cl(C). By hypothesis, A ⊆ B. Therefore C ⊆ B. Since A ⊆ cl(C), cl(A) ⊆ cl(C) and B ⊆ cl(C), by hypothesis. Hence by [10], B is an IFSPOS. As every IFSPOS is an IFβGOS by Theorem 3.2, B ∈ IFβGO(X).

Theorem 3.20: Let (X, τ) be an IFTS and A, B ⊂ X, If B is IFβGO(X) and βint(B) ⊂ A then A ∩ B is IFβGO(X).

Proof: Since B is IFβGO(X) and βint(B) ⊂ A, βint(B) ⊂ A ∩ B ⊂ B, by Theorem 3.15, A ∩ B is IFβGO(X).

CONCLUSION
Thus we have analysed the relationship between intuitionistic fuzzy β generalized open sets and the already existing intuitionistic fuzzy open sets and obtained many interesting theorem concern with the intuitionistic fuzzy β generalized open sets.

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