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MODELING OF VOLATILITY OF INTEREST AND TREASURY BILL RATES USING ARCH / GARCH FAMILY MODELS AND THEIR EFFECT ON PENSION FUND

Isaac Onchaga Ondieki* and Professor Simwa
*Department Of Actuarial Science, School Of Mathematics, University Of Nairobi

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ABSTRACT
Frequent fluctuations in interest rates often affect purchasing power of citizens and many businesses. Prices of commodities rise upwards very fast in most cases but take time drop. If these fluctuations are predicted early enough, probably measures can be taken in advance to deal with these shocks before the prices of commodities change. In this study we model the volatility of interest rates and Treasury bill rates using GARCH family models and asses how each of them affect the growth of pension fund.

The interest and Treasury bill rates were converted into simple returns and modeled by use of ARCH and GARCH (1, 1) models. The GARCH(1, 1) was preferred because it allows many parameters and considers conditional heteroscedasticity of data to assess volatility of interest rates and Treasury bill rates. Volatility measures the errors made in modeling returns. It was discovered that the average volatility is not constant but varies with time and can be forecast or predicted in both cases. A multifactor modelwas used to investigate how each affect pension Fund, it was revealed that interest rates affected pension fund more than the Treasurybill rates, and the model can be used to project growth of the fund.

INTRODUCTION

Pension fund
Pension Fund was established by the Government of Kenya through the act of Parliament and was last revised in 2009.It provided for regulation and granting, gratuities and other benefits to public service officers employed by the government of Kenya

The Government of Kenya also established Retirement Benefits Authority to regulate, supervise and promote retirement benefit schemes for private employers and their employees. They make contributions to this scheme. However, formal employers and workers are free to join this scheme. The benefits can be paid as long as the employee has worked for a period of ten years or above upon retirement or death whichever comes early.

The contributions are invested to earn interest so that the beneficially is paid principal plus interest earned. The money from this fund is invested in Government Treasury bills and other securities. The interest and Treasury bill rates changes are expected to affect the returns on this fund.

Interest Rates
Investment firms such as insurance companies and pension fund, invest received premiums, largely in fixed income securities such as Treasury bills and Bonds. Changes in interest rates often affect returns from these securities over a period of time. The value of assets and liabilities of investment firms change as interest rates change and thereby expose the company to risk.

This study focuses on interest rate environment and changes in Treasury bill prices and their effect on Pension Fund. The purpose is to establish whether there exists a relationship between interest rates changes, Treasury bill rates and the pension Fund. The suitable and appropriate Models, are required in analyzing the effects of interest rates changes and the changes on Treasury bill rates.

The use of stochastic processes and time series models are found to be appropriate in forecasting when applied in financial data in Economics hence we will consider some of these models in this study.

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Many countries in the world experience fluctuations in interest rates more often. For example in the USA in2012, the country experienced low interest rates that threatened the insurance industry according to Economic perspective, KyalBerends*et al*(2013), In their research, noted that when interest rates fall, bond prices rise, also that some of the products like annuities may not be sold easily when interest rates are very low. The effect was more felt in large insurance firms than the small firms. In the USA interest rates swap derivatives were used to hedge changes in interest rates (Economic perspective, 2013)

In this study GARCH model (1, 1) is used to analyze the volatility in interest rates (bank lending spot rates) and Multifactor models are used to establish any relationship between interest rates, changes in rates of treasury bills and the pension fund. The two models are good in analyzing financial data that vary with time. These models are used mainly in time series applications.

Treasury Bills
A treasury bill is a paperless short-term borrowing instrument issued by the Government to raise funds from the public or institutional investors. Treasury bills are issued in maturities of 91, 182 and 364 days. They are sold at a discounted price to reflect investor's return and redeemed at face (par) value. The difference between discounted value and par value represents the rate of return to the investor. Any investor is required to have an active Central Depository System (CDS) account with the CBK so that it is easy to trade in securities. The discounted price of the Treasury bill depends on the interest rate/yield quoted by the investor and is calculated using the following formula:

$$p = 100\left[\frac{1}{\left(1+\left(\frac{r}{100}*\frac{d}{365}\right)\right)}\right](1)$$

Where;
p-price per ksh.100
r -interest rate
d -Days to maturity

In this study, we only focus on Treasury bill rates whose maturity is 91 days.

The Treasury bill and bank interest rates vary widely from one period to the next. There are a number of factors that cause these variations. Theoretically interest rates also depend on a range of factors such as inflation, state of economy of the country, balance of payments and etc.Investments done by companies are likely to be affected bythese variations making it difficult to accurately predict future profits. Returns from investments in Treasury bills can be predicted if the interest rates are fairly constant although this is not the case. Treasury bills have a short period than Bonds that take long period to mature, and the interest rates do not remain constant over this period. The knowledge of volatility of interest ratesis very important in deciding the best investment portfolios for the investment companies in any given period.

A research done by Boubaker&Sghaier (2011), effect of interest rate and inflation rate on non-life insurance premiums, showed that both have an impact on non-life insurance premiums which depended on the value of the inflation rate. They used Panel Smooth transition Error Correction Model (PSTECM) in analysis, the model took into account both short and long-run effects of changes in economic variables.

Research done by KyalBerends et al (2010) on sensitivity of insurance firms to interest rates changes showed that fluctuations in interest rates affected life insurance business either positively or negatively. Stock price changes was analyzed with those of interest rates changes. Prices were taken at corporate level involving many firms other than the individual firms in USA.In this research a two-factor Model was used that was suggested by Brewer, Mondschean, and Strahan (1993) and James (1984).

Volatility of interest rates can be studied by applying ARCH and GARCH models, which are appropriate in describing volatility in financial data with times series characteristics, Engle,(1992).The GARCH family models are good in capturing homoscedasticity and Heteroscedasticity in financial data.GARCH (1, 1) model is more adequate in describing most financial time series Boleslaw et al, (1992)

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The data where variances are not equal, suffers from what is called heteroscedasticity.

The GARCH models treat heteroscedasticity as a variance to be modeled, this mostly has been applied to non-time series models to find the standard errors to reduce heteroscedasticity.
For large sample size data concern of heteroscedasticity is minimal or less significance however for time series it can be noted that some periods are riskier than others although not scattered randomly. The GARCH model which stand for Generalized Autoregressive Conditional Heteroscedasticity is applied in dealing with these situations mentioned above.

McNeil et *al* (2000) proposed a two stage model where GARCH model is fitted to return data and used to model the tail of residuals using EVT Model. EVT models may be used to model the risk of extreme values or rare events. The challenge is where extreme data is scarce. There is a challenge in choosing the best method of estimating the parameters.

Bollerslev (1986) gave a generalized model of GARCH of estimating these parameters. It proved to have yielded successful results in predicting conditional variances. Both interest rates and Treasury bill rates vary with time, this method can be used to determine the variance in each case.

In a study carried by Papadamou and Siriopoulos (2014) in UK, showed that fluctuations of interest rate affected significantly stock returns of companies. They investigated effect of Monetary Policy committee (MPC) on interest rate risk and insurance companies in UK. CAPM and Fama-French GARCH-models were used in modeling the interest rates and stock returns.

Investigation done by Mohammed Torkestani and ElhamBorujerdi (2014), found that there is a positive relationship between rates charged by life insurance firms and the bank rates in Iran, that's fluctuations occurring in bank rates resulted to changes in rates charged by insurance firms.

Maina,kamau and Kasungu(2013) noted in their research that the key component in foreign exchange market stability is political stability of a country otherwise high volatility in exchange rates will force firms to add risk premium to prices of their products. Usually measures taken by central Bank to maintain exchange rate stability have an effect on bank interest rates which directly or indirectly affect the prices of Treasury bills.

The objectives of this study in threefold; First is to assess the volatility of interest rates, Second the volatility of Treasury bill rates using ARCH/GARCH models in each case and Lastly how the changes in interest rates and Treasury bill rates each affect the growth of pension fund.

In section 2.1, we shall deal with brief description of ARCH model, its properties, how to apply it in financial data and its weaknesses. In section 2.2, we shall briefly describe GARCH (1,1) model, its properties, how to fit in financial data and its weaknesses. Section 2.3 we deal with Conditional error distribution that is used in the GARCH (1,1) model. Section 2.4 we focus on model used to assess the effect of interest rate and Treasury bill rates on Pension fund, Section 2.5, we discuss the results and analysis, then the conclusion.

The ARCH MODEL

The ARCH known as (Auto-regressive Conditional Heteroskedastic model) was introduced by Engle (1982), which allows the conditional variance to change over time and can be expressed as a function of past errors. The unconditional variance is the standard measure of the variance, When it remains constant it is called Homoscedastic and ordinary least squares are used to estimate α and $\beta$of the standard linear regression formula. The conditional varianceσ_t is the measure of our uncertainty about a variable y_t given a model and information set ψ_t and is expressed as a linear function of past sample variance only. If the variance of the residual is not constant, it is called Heteroskadasticitic and weighted least squares is used to estimate the regression coefficients.

Suppose that$y_{1,}$ y_2 … $y_t$time series observations and let ψ_t be the set of y_t observations up to time t, for $t \le 0$. The process $\{y_t\}$ is an Autoregressive Conditional Heteroscedastic process of order p, ARCH (p), if:

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$r_t = \mu + y_t$ (2)

$y_t = \sigma_t \varepsilon_t \;\; \varepsilon_t \sim N(0, \sigma^2)$ (3)

$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \alpha_j y_{t-j}^2$ (4)

With $\alpha_0 \geq 0$, $\alpha_j \geq 0$ and $\sum_{j=1}^{p} \alpha_j < 1$ to ensure positive variance and $\alpha_1 < 1$ for stationarity are ARCH model parameter limits, where ε_t error term follows normal distribution and is i.i.d random variables.

Properties of ARCH (p) Model

1. **The Mean;**

From equation 3, the conditional expectation and variance of $x_t$is, given the expectations of ε_t is zero, then the expectations of y_t is given as;

$E(y_t) = 0$

2. **The Second Moment or Variance;**

$E(y_t^2) = E(\sigma_t^2 \varepsilon_t^2) \; E(\sigma_t^2)$ (5)

Since $\sigma^2 = 1$,following a standard normal distribution of ε_t .then

$E(\sigma_t^2) = \alpha_0 + \alpha_j \sum_{j=1}^{p} E(y_{t-1}^2)$ (6)

Given $E(\sigma_t^2) = E(y_{t-1}^2)$ under stationarity assumption,

$E(\sigma_t^2) = \dfrac{\alpha_0}{1 - \sum_{j=1}^{p} \alpha_j}$ (7)

For ARCH (1), when j = 1, the expectation of conditional squared variance is given by;

$E(\sigma_t^2) = \dfrac{\alpha_0}{1 - \alpha_1}$ (8)

3. **The Kurtosis**

Is a measure of the extent to which observed data fall near the Centre of the distribution or the tails. It is a normalized form of the fourth central moments of the distribution determined as follows;
First the forth moment of the time series is obtained as;

$$E(y_t^4) = E\{(\sigma_t^2)^2 \varepsilon_t^4\} = E\{(\sigma_t^2)^2\} E(\varepsilon_t^4)$$

$$= 3E\{(\sigma_t^2)^2\} \text{ (9)}$$

$$E\{(\sigma_t^2)^2\} = E\{(\alpha_0 + \sum_{j=1}^{p} \alpha_j y_{t-1}^2)^2\}$$

$$= \alpha_0^2 + 2\alpha_0 \sum_{j=1}^{p} \alpha_j E(y_{t-1}^2) + \sum_{j=1}^{p} \alpha_j^2 E(y_{t-1}^4)$$

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Substituting equation 9, we have;

$$E(y_t^4) = 3\{\alpha_0^2 + 2\alpha_0 \sum_{j=1}^{p} \alpha_j E(y_{t-1}^2) + \sum_{j=1}^{p} \alpha_j^2 E(y_{t-1}^4)\}$$

Under stationarity, $E(y_{t-1}^2) = E(\sigma_t^2) = \dfrac{\alpha_0}{1 - \sum_{j=1}^{p} \alpha_j}$

and since $E(y_t^4) = E(y_{t-1}^4)$ then gives us

$$E(y_t^4) = 3\frac{\alpha_0^2(1 + \sum_{j=1}^{p} \alpha_j)}{(1 - \sum_{j=1}^{p} \alpha_j)(1 - 3\sum_{j=1}^{p} \alpha_j^2)} \quad (10)$$

The Kurtosis is given by;

$$= \frac{E(y_t^4)}{\{E(y_t^2)\}^2}$$

Substituting equations 8 and 10, we get;

$$3\frac{(1 + \sum_{j=1}^{p} \alpha_j)(1 - \sum_{j=1}^{p} \alpha_j)}{1 - 3\sum_{j=1}^{p} \alpha_j^2} \quad (11)$$

Therefore, the kurtosis is

$$= 3\frac{1 - \sum_{j=1}^{p} \alpha_j^2}{1 - 3\sum_{j=1}^{p} \alpha_j^2} \quad (12)$$

When j=1, we get ARCH(1), then the Kurtosis of ARCH(1) is;

$$= 3\frac{1 - \alpha_1^2}{1 - 3\alpha_1^2} \quad (13)$$

Which is strictly greater than 3 unless $\alpha_1 = 0$. The kurtosis for a normally distributed random variable Z is 3. Thus, the kurtosis of y_t is greater than the kurtosis of a normal distribution, and the distribution of y_t has a heavier tail than the normal distribution, when $\alpha_1 > 1$ often described as Leptokurtic distribution which has a high peak with heavy tail.

Fitting Procedure for ARCH model

There are two steps in fitting in the ARCH Model:

1st step: Plot the return of interest rates with time and Treasury rates with time, Log-returns of both interest rate and treasury rates and analyze the autocorrelation function (ACF) and the partial autocorrelation function (PACF) between them.

An ARCH model assumes working with returns, therefore variables are to be converted into returns.

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2nd step: Performtests, such as ARCH effect test or the Q-test.

Autocorrelation detected has to be quantified. Quantification is done by the preceding the Ljung-Box-Pierce Q-test and Engle's ARCH test. Performing a Ljung-Box-Pierce Q-test, it can be verified approximately, the presence of any significant correlation in the returns when tested for up to 20 lags of the ACF at the 0.05 level of significance.

The tool for analysis used is the R-studio, which was downloaded from the internet and the packages of R from the Crawn loaded into it.The variable data is imported from excel into R.The data is run and results obtained are discussed in section 3.

Weakness of ARCH model

Despite ARCH model able to capture the characteristics of financial time series data, it has some weaknesses which include; ARCH treats positive and negative returns in the same way (by past square returns) andis restrictive in parameters. It often over-predicts the volatility, because it responds slowly to large shocks and volatility from it persists for relatively short amount of times unless p is large.

The GARCH Model

Anew model was developed to address the weaknesses of ARCH model.Bollerslev (1986) proposed a useful extension of ARCH; the Generalized Autoregressive Centralized Heteroskedastic Model (GARCH) with only three parameters that allow for an infinite number of squared roots to influence the current conditional variance unlike the ARCH. It provides relatively long lag in the conditional variance equation. This feature allows GARCH to be more persistence than ARCHmodel.GARCH process allows lagged conditional variances to enter in the model (Bollerslev 1986), it also avoids negative variance parameters for fixed lag structure.

Although ARCH incorporates the feature of Autocorrelation observed in return volatility of most financial assets, GARCH improves ARCH by adding a more general feature of conditional heteroscedasticity. Parameters *p* and *q* in GARCH (*p, q*) are chosen to capture significant spikes in the autocorrelation function, they are frequently used for modeling the volatility of financial returns. These models generate good estimates with few parameters.

The process y_t is a Generalized Autoregressive Conditional Heteroscedastic model of order p and q, GARCH (p, q) if:

$r_t = \mu + y_t$ And

$y_t = \sigma_t \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2)$ therefore

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + ... + \alpha_p y_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2$$

$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (14)$$

Where q>0, $p \geq o, \alpha_0 > 0, \alpha_i \geq 0$ for i =1, 2… p, $\beta_j \geq 0$ for j=1…q are the GARCH model parameter limits. Again these conditions are needed to guarantee that the conditional variance $\sigma_t^2 > 0$ is positive. The term α_0 is constant. The ARCH term y_{t-i}^2 [eq.14] is the first lag of the squared residual from the mean equation and represents volatility from the previous period and the GARCH term σ_{t-j}^2 [eq.14] represents last period's lag of the forecast variance.

Properties of GARCH (p, q),

This model has the following properties that make it suitable in modelling volatilities of financial data

1. **The mean;**

From equation 14, the conditional expectation and variance of x_t is:

$E(y_t) = 0$, since the expectation of ε_t is 0.

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2. The Second Moment or Variance;

$$E(y_t^2)=E(\sigma_t^2\varepsilon_t^2)=E(\sigma_t^2)$$

$$E(\sigma_t^2)=\alpha_0+\sum_{i=1}^{p}\alpha_i E(y_{t-i}^2)+\sum_{j=1}^{q}\beta_j E(\sigma_{t-j}^2) \quad (15)$$

Given $E(\sigma_t^2)=E(y_{t-1}^2)=E(\sigma_{t-j}^2)$ under stationary assumption,

$$E(\sigma_t^2)=\frac{\alpha_0}{1-(\sum_{i=1}^{p}\alpha_i+\sum_{j=1}^{q}\beta_j)} \quad (16)$$

For GARCH (1, 1) when j = 1, then

$$E(\sigma_t^2)=\frac{\alpha_0}{1-(\alpha_1+\beta_1)} \quad (17)$$

3. The Kurtosis;

It is obtained from the fourth moment as explained in section 2 part (iii) as follows;
First the fourth moment of the time series is obtained;

$$E(y_t^4)=E\{(\sigma_t^2)^2\varepsilon_t^4\}=E\{(\sigma_t^2)^2\}E(\varepsilon_t^4)=3E\{(\sigma_t^2)^2\}$$

But

$$E\{(\sigma_t^2)^2\}=E\{(\alpha_0+\sum_{i=1}^{p}\alpha_i y_{t-i}^2+\sum_{j=1}^{q}\beta_j\sigma_{t-j}^2)^2\}$$

$$=\alpha_0^2+2\alpha_0\sum_{i=1}^{p}\alpha_i E(y_{t-i}^2)+2\alpha_0\sum_{j-1}^{q}\beta_j E(\sigma_{t-j}^2)+\sum_{i=1}^{p}\alpha_i^2E(y_{t-i}^4)+\sum_{j=1}^{q}\beta_j^2E[(\sigma_{t-j}^2)^2]+2\sum_{i=1}^{p}\sum_{j=1}^{q}\alpha_i\beta_j E(y_{t-i}^2\sigma_{t-j}^2)$$

When$i = j = 1$, we get GARCH (1, 1)

$$E\{(\sigma_t^2)^2\}=\alpha_0^2+\alpha_1^2E(y_{t-1}^4)+\beta_1^2E\{(\sigma_{t-1}^2)^2\}+2\alpha_1\beta_1E(y_{t-1}^2\sigma_{t-1}^2)+2\alpha_0\alpha_1E(y_{t-1}^2)+2\alpha_0\beta_1E(\sigma_{t-1}^2)$$

$$=\alpha_0^2+(3\alpha_1^2+2\alpha_1\beta_1+\beta_1^2)E\{(\sigma_{t-1}^2)^2\}+2\alpha_0(\alpha_1+\beta_1)E(\sigma_{t-1}^2) \quad (18)$$

Assuming the process is stationary, $E\{(\sigma_t^2)^2\}=E\{(\sigma_{t-1}^2)^2\}$

Hence

$$E\{(\sigma_t^2)^2\}=\frac{\alpha_0^2+2\alpha_0(\alpha_1+\beta_1)E(\sigma_{t-1}^2)}{1-3\alpha_1^2-2\alpha_1\beta_1-\beta_1^2}$$

$$=\frac{\alpha_0^2+2\alpha_0^2(\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)(1-3\alpha_1^2-2\alpha_1\beta_1-\beta_1^2)}$$

$$E(y_t^4)=3E\{(\sigma_t^2)^2\}$$

$$=3\frac{\alpha_0^2+2\alpha_0^2(\alpha_1+\beta_1)}{(1-\alpha_1-\beta_1)(1-3\alpha_1^2-2\alpha_1\beta_1-\beta_1^2)} \quad (19)$$

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The Kurtosis is given by;

$$= \frac{E(y_t^4)}{\{E(y_t^2)\}^2}$$

Substituting equation 18 and equation 19, we get;

$$= 3\frac{1-(\alpha_1+\beta_1)^2}{1-3\alpha_1^2-2\alpha_1\beta_1-\beta_1^2} \tag{20}$$

Which is strictly greater than 3 unless $\alpha_1 = 0$, Again under GARCH (1,1) shows that the distribution has a heavy tail and high peak than the normal distribution.

The same fitting procedure is applicable for a general GARCH (p, q) as the ARCH fitting above.

Forecast of Conditional Variance in GARCH model

The formula used to calculate the multi-step forecasts of the conditional variance for the GARCH (1, 1) model is illustrated below, the variance equation is

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1\sigma_{t-1}^2 \tag{21}$$

Denote the forecast origin by n and the forecasted value by h andlet F_n be the information set available at time n. For $h = 1$, the 1-step ahead forecast of the conditional variance is simply

$$E(\sigma_{n+1}^2 | F_n) = E(\alpha_0 + \alpha_1 y_n^2 + \beta_1\sigma_n^2 | F_n)$$
$$= \alpha_0 + \alpha_1 y_n^2 + \beta_1\sigma_n^2 \tag{22}$$

For $h = 2$, 2-step then, it becomes

$$E(\sigma_{n+2}^2 | F_n) = E(\alpha_0 + \alpha_1 y_{n+1}^2 + \beta_1\sigma_{n+1}^2 | F_n)$$
$$= \alpha_0[1+(\alpha_1+\beta_1)] + \alpha_1(\alpha_1+\beta_1)y_n^2 + \beta_1(\alpha_1+\beta_1)\sigma_n^2 \tag{23}$$

Recursively, it is easily seen that for $h = j$, the j-step ahead forecast of the conditional variance of the GARCH (1, 1) model is;

$$E(\sigma_{n+j}^2 | F_n) = \alpha_0\sum_{k=o}^{j-1}(\alpha_1+\beta_1)^k + \alpha_1(\alpha_1+\beta_1)^{j-1}y_n^2 + \beta_1(\alpha_1+\beta_1)^{j-1}\sigma_n^2$$
$$= \alpha_0 + (\alpha_1+\beta_1) * E(\sigma_{n+j-1}^2 | F_t) \tag{24}$$

Therefore, the forecasts of the conditional variances of GARCH (1, 1) model can be computed recursively.

The GARCH (1,1) model is good in predicting volatility changes .The model describe the time evolution of the average of squared errors i.e. magnitude of uncertainty however they fail to explain why uncertainty tends to cluster. It does well in some periods and worse in others. The interest rates are modeled to determine the moderate and peak periods of changes using volatility. The same is applied to changes in rates of Treasury bills.The model is suitable in symmetrical distribution but fails in asymmetrical distribution. Other models are developed to deal with this problem such as EGARCH.

Conditional Error Distributions

The following error distributions are analyzed to select the best distribution to use in forecasting in GARCH (1, 1)**.**

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(i) Normal Distribution
The probability density function is given as;

$$f(x;\ \mu,\ \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ,-\infty < x < \infty$$

(ii) Student- t Distribution
When $\nu \to \infty$ the distribution converges to a standard Normal
The probability density function is given as;

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)\left(1+\frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}}$$

(iii) Generalized error distribution whose probability density function is

$$f(x) = \frac{\lambda.s}{2.\Gamma\left(\frac{1}{s}\right)} . \exp(-\lambda^s . [x-\mu]^s \quad (3.5.15)$$

Where,
λ –scale parameter
μ-location parameter
$\mu(z)$-Euler Function
s- Shape Parameter.

Model selection criteria
The distributions discussed above may not be applied all at the same time in GARCH (1,1) ,They are also expected to yield nearly same results but there's a need to use the one that gives desirable result. Selection criteria is used to find out whether the fitted distribution model gives an optimum balance between persistence and goodness-of-fit. The common models used for criteria selection are AkaikeInformation Criterion (AIC), Hannan- Quinn criterion (HQ), Bayesian Information criterion (BIC), Schwarz Information criterion (SIC), and Log likelihood criterion.

Equation of each model is as:
AIC = -2log (maximum likelihood) + 2k, where k= p+q +1 if the model contains an intercept or a constant term, k = p+q.

BIC = -2log (L) +2(m)
HQ = -2log (L) + 2mlog (log (n)).
SIC = -2log (L) + (m + mlog (n))
Log (L) is the log likelihood.
The model that gives the minimum or lowest value is the most desirable to use.

MUILTFACTOR MODEL
This is a financial model that employs multiple factors in its computations to explain market phenomena or equilibrium asset prices. The multifactor model can be used to explain either an individual security or a portfolio of securities. It compares two or more factors to analyse relationships between variables and the security's resulting performance.

The multifactor model was considered for this study as the variables are mainly macro-economic and financial data. These variables have unique characteristics that can only be handled by theP_i the performance on the pension Fund

a_i, $c_i$are the constant and random parts respectively of the component of pension fund performance

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I_1 ... $I_4$are the systematic economic factors that influence the performance of pension fund obtained from the GARCH forecasts. The volatility of the individual variables which were modelled using the GARCH model were consolidated to give the overall pension plan performance.
A multifactor model for the pension plan performance is given by an equation of the form:

$$P_i = a_i + b_{i,1}I_1 + b_{i,2}I_2 + b_{i,3}I_3 + b_{i,4}I_4 + c_i \quad (26)$$

Where:
$b_{i,4}$ is the sensitivity estimates of the economic variables, that is they represent the standard deviations of variables which were obtained after the factors have been modelled.

The following are the assumptions of Multifactor model:
The factor realizations, I_t are stationary with unconditional moments

$$E(I_t) = \mu_I$$
$$\text{cov}(I_t) = E\{(I_t - \mu_I)(I_t - \mu_I)'\} = \Omega_I \quad (27)$$

The specific error term c_{it} , are uncorrelated with each of the common factors, I_{ki} ,

$$\text{cov}(I_{kt}, c_{it}) = 0 \text{ for all } k, i \text{ and t}$$

Error terms c_{it} are serially uncorrelated and contemporaneously uncorrelated across assets

$$\text{cov}(c_{it}, c_{js}) = \sigma_t^2 \text{ For all } i = j \text{ and } t = s$$

(i) Fitting Multi-factor model
The following are the steps required to fit the multifactor model;
Step1: Obtain the values of $b_i's$ of the variables using GARCH model as the standard deviations of these variables.
Step2: Forecast the economic variables $I_i's$ using GARCH model
Step3: Linearly combine these values to obtain a multifactor model with a_i being the base value or constant term and c_i the random variable error term with a zero error mean.

DATA ANALYSIS, RESULTS AND DISCUSSION
This section deals with the analysis and interpretation of the results of monthly interest rates and Treasury bill rates volatilities from January 2012 to July 2016.

Descriptive Statistics
The raw data of interest rates considered in this study is the weighted average for each month from January 2012 to July 2016.

As already mentioned earlier the interest rate is converted to returns before using the ARCH/GARCH models. Similarly the Treasury bill rates will be analyzed using the same formula for returns.

$$l_t = \left(\frac{p_t}{p_{t-1}} - 1\right) * 100\%$$

Where
p_t -The current interest rate
p_{t-1} –The previous interest rate
Formula for returns is given as:

$$r_t = \log \frac{l_t}{l_{t-1}}$$

Where r_t is returns up to period t

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Raw data analysis

- The time plot of the monthly interest rates against time is as shown in figure 1;

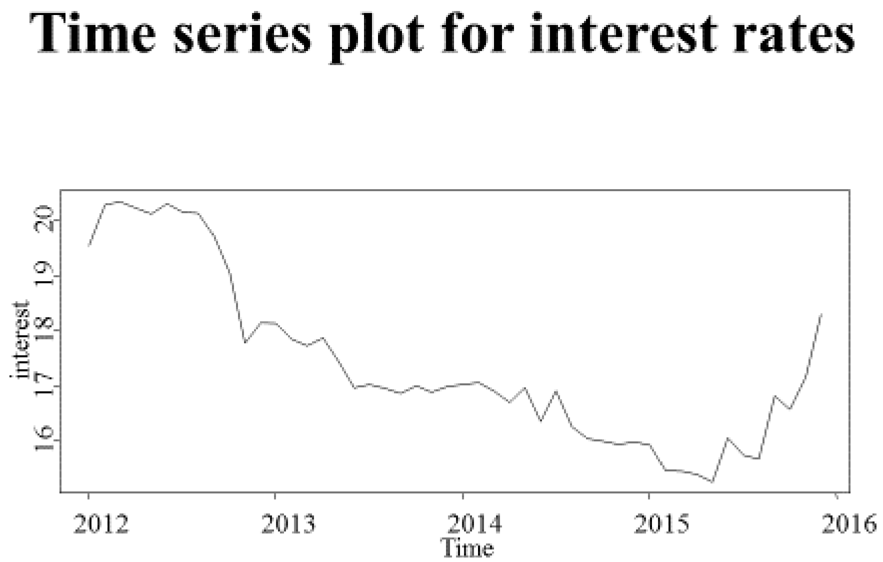

Figure 0 1: raw data plot of interest rates vs. Time in months

The interest rate trend can be seen clearly, gradually decreasing from peak at December2012 to February 2015 and starts rising again to July 2016. The volatility characteristics of financial time series data can be clearly seen from the fall –rise of interest rates

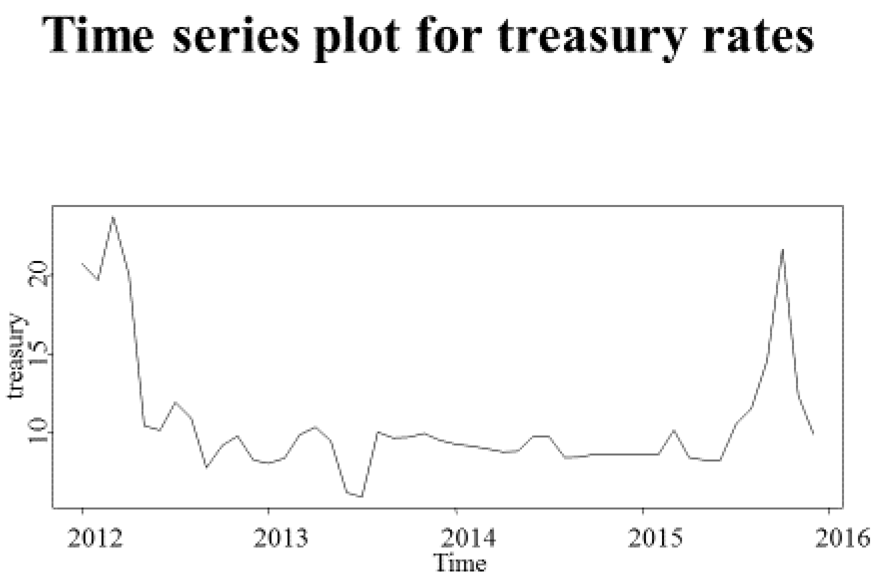

Figure 0 2: plot for treasury bill rates vs. Time (months)

The trend of Treasury bill rates is clearly falling from peak December 2012 to the middle of the year and fairly stabilizes up to April 2015, it rises again sharply towards July 2016. Volatility characteristic of financial time series data is clearly seen. In both cases it is observed that highest rates occur at the same period .It shows that there is a correlation between interest rates and Treasury bill rates which need further investigation.

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Time-series characteristics for both plotted drawn combined.

The plotclearly shows variation of interest rates with time, peak period and low periods occurring at the same showing correlation features. Both rates are time dependent or display characteristics of time-series financial data.

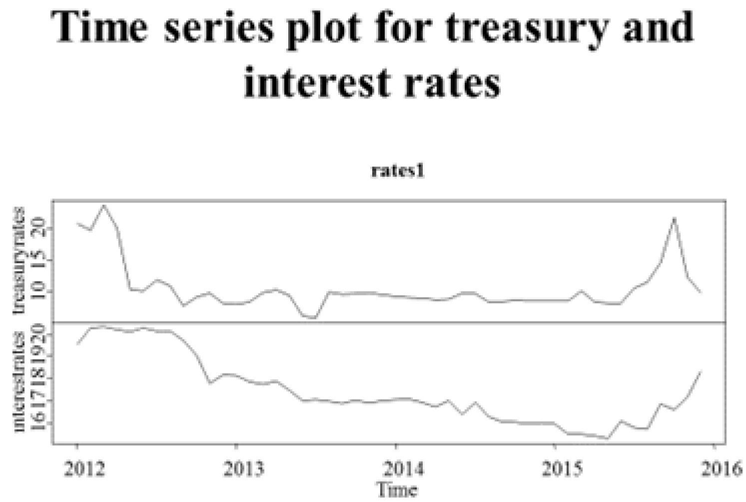

Figure 0 3: combined plot for interest rates and treasury bill rates

Peak periods are 2012 and 2016 when rates were very high. We can proceed to the autocorrelation and partial correlation

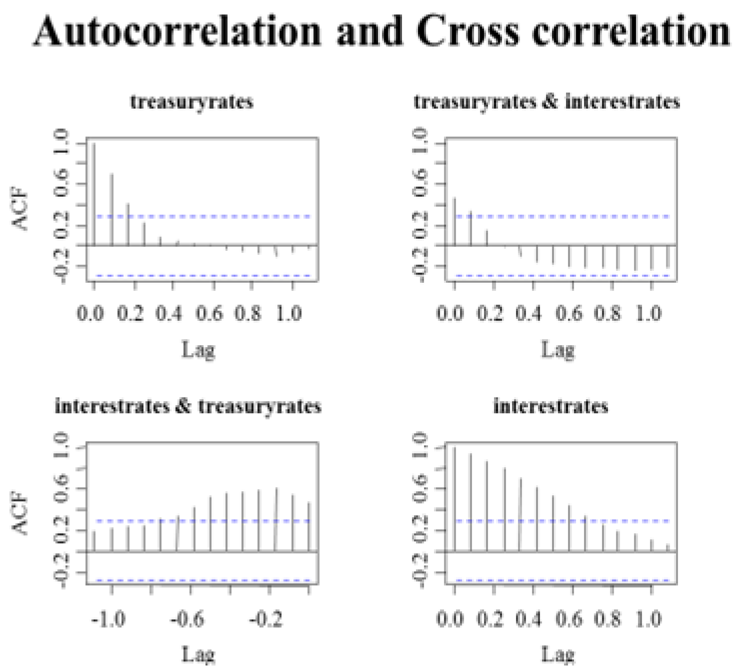

Figure 0 4: the autocorrelation and cross-correlation of interest and treasury rates

Key:
Blue line for Interest rates
Dark line for Treasury rates

ARCH EFFECT TESTS

We can show the ARCH effects by using simple and log returns, log interest rates and Treasury rates.

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Simple and Log returns Descriptive Statistics

Table i-1: Simple and Log Return interest and Treasury rates

	Interest	Treasury
nobs	**54.000000**	**54.000000**
NAs	**0.000000**	**0.000000**
Minimum	**-0.066176**	**-0.478443**
Maximum	**0.072704**	**0.695058**
1. Quartile	**-0.011587**	**-0.078151**
3. Quartile	**0.005225**	**0.047109**
Mean	**-0.001140**	**-0.000985**
Median	**-0.003751**	**-0.006121**
Sum	**-0.061540**	**-0.053171**
SE Mean	**0.003249**	**0.025909**
LCL Mean	**-0.007656**	**-0.052952**
UCL Mean	**0.005377**	**0.050982**
Variance	**0.000570**	**0.036249**
Stdev	**0.023874**	**0.190392**
Skewness	**0.747587**	**0.658052**
Kurtosis	**3.074789**	**3.906281**

Based on the results of basic statistics of data, mean of simple and log return for interest and Treasury rates are -0.001140 and -0.000985 respectively are very close to zero. The values of kurtosis are 3.074789 and 3.906281 which is greater than 3 hence the data exhibits excess kurtosis showing heavy tailed distribution. The values of skewness are 0.747587 and 0.658052 for interest rate and Treasury rates respectively which are greater than zero showing that the distribution is not symmetrical.

Standardized Residuals Tests

Jarque-Bera Test R Chi^2 = 25.11513statistic-p-value = 3.518182e-06

Table i-2: Ljung box test for log return for interest and Treasury rates.

	Interest	Treasury
Test statistics	10.52586	13.91997
Parameter	12	12
p-value	0.5699285	0.305848

From Ljung box test for log returns, p-value and test statistics suggest that ARCH effects are significant since p-value are less than one

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Plot for log return of Treasury rates From ARCH Model

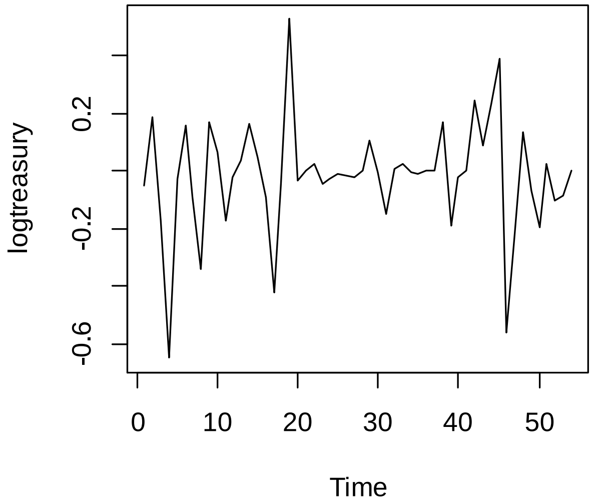

Figure i-1: Log returns of treasury bills vs. time in months

From the log returns plots of returns, volatility clustering can be clearly seen where there's a fall-rise or rise-fall of rates.. The mean reverting property can also be seen clearly where the returns revolve around a certain value. ARCH characteristics is shown from the figure, the fact it shows both negative and positive values of log returns.

Garch Model

The square log plots for interest rates and Treasury rates are shown the figures below,

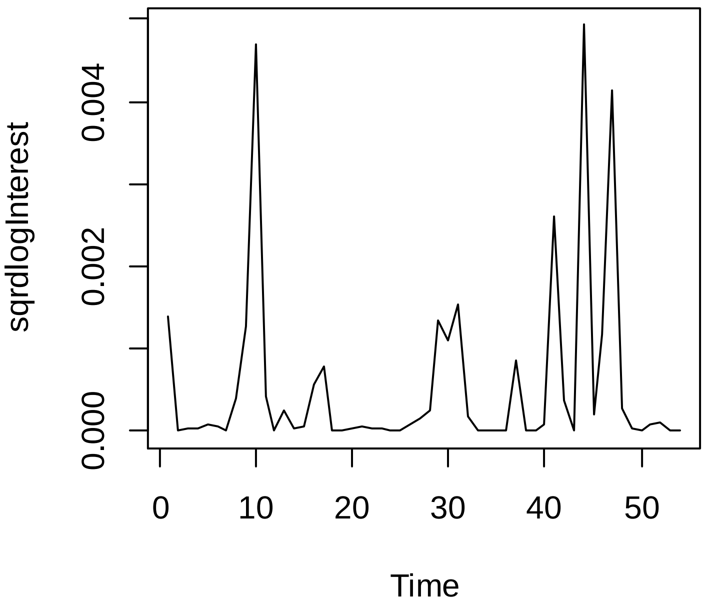

Figure i-2: plot of log returns of interest rates

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It is clearly seen from the plots GARCH effects differ from Arch due to fact that it treats returns as positive only. Volatility is again clearly shown, for interest rates are very high in the 10^{th} and between 40^{th} and 50^{th} months. The advantage of GARCH model is that it treats all returns as positive**.**

Distributions Models Selection Criteria

NORM,Sd-t, and GED

There are three distributions suggested, thus Normal distribution, Student T test and GED ,we use the following formulas to test their suitability before it is used in the GARCH (1,1) modeling of the interest rates and Treasury bill rates changes. We can use the QQ plots to see which one tries to fit the data in linear model.

The results of QQ plots is as shown below;

QQ plot for GARCH(1,1),Normal Distribution

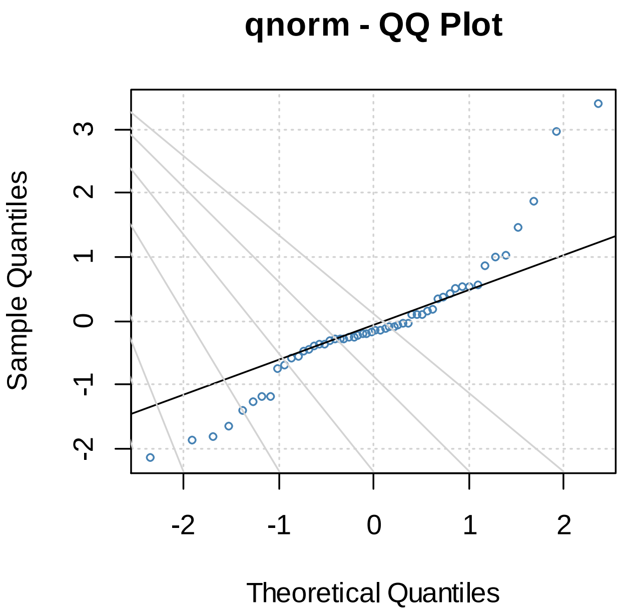

Figure i-3: QQ plot of GARCH (1,1) with normal Distribution

This is clearly indicated by the failure of the data to be linear at the trails, it suggests a heavily tailed distribution for the residuals since norm QQ plots poorly fits.

QQ plots for GARCH(1, 1), student T distribution.

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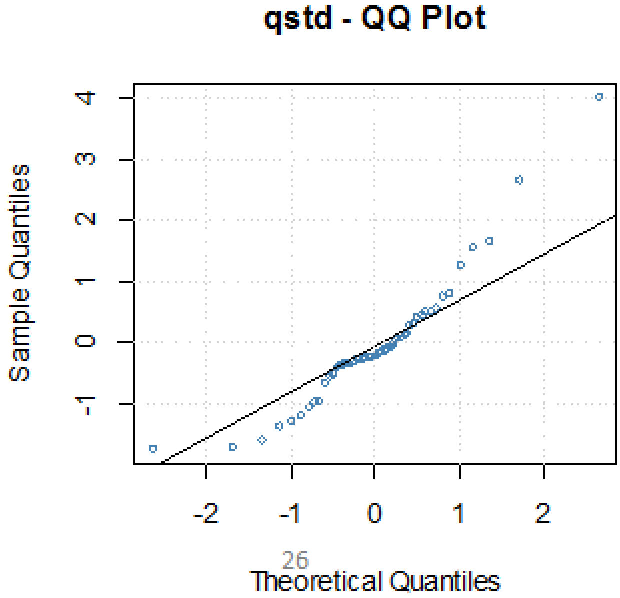

Figure 0 8: qq Plots For Garch (1, 1), Student t Distribution

The std-QQ plot seems to have relatively fair fits with student-t distribution being the residual distribution QQ plot for GARCH(1, 1), GED distribution.

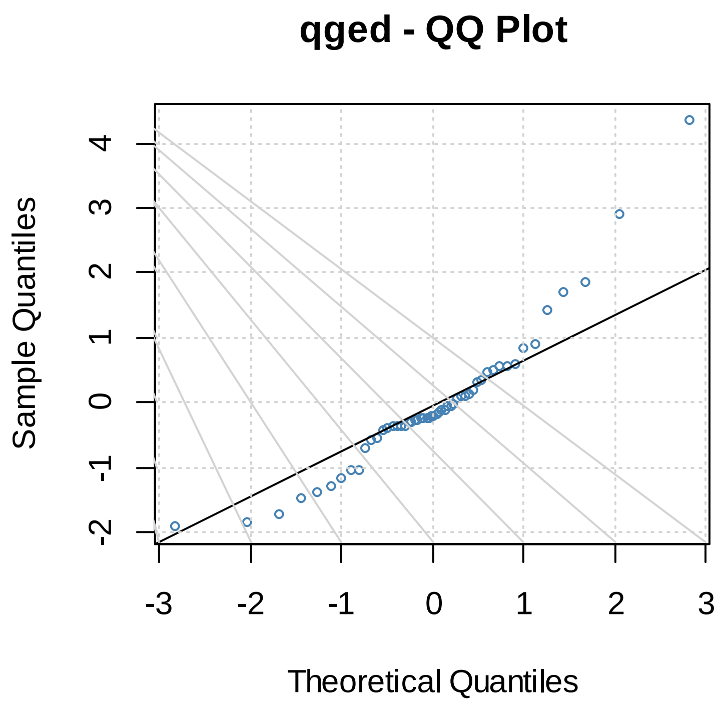

Figure i-4: QQ plot for GARCH (1, 1), GED distribution

It is clearly that the GED-QQ plot fairly fit than the student-t distribution. The data fits along the line.it seems that three distributions fits the data into linear model, however GED stands out to be the best where the gap between the line and fitted data is very small as compared to the other two. Based on this information, now the GED distribution with GARCH (1, 1) is used to predict the future volatility of interest and Treasury rates.

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Testing using AIC, BIC, SIC, HQIC and LL

The tests showed the following results, Criterion Statistics:

Table i-3: Criterion statistics

AIC	BIC	SIC	HQIC	LL
-4.646305	-4.529806	-4.64O056	-4.597690	128.2882

The criteria used in selecting the model is the one with minimum orsmallest value. It is evident from the figures that the lowest is -4.646305 which is the AIC.

COMPARISON OF AIC OF NORMAL DIST., STUDENT T, AND GED;

We perform yet another test along with the condition α +β < 1 for GARCH for each distribution to choose the best distribution to be used in GARCH (1, 1) model. The results are displayed in the following tables shown below;

StandardizedResiduals Tests Statistic p-Value

Jarque-Bera Test RChi^2 25.10385 3.538088e-06
Shapiro-Wilk Test R W0.9045178 0.000407079
Ljung-Box Test Q(10) 12.90447 0.2290624
Ljung-Box Test R Q(15) 19.03649 0.2120849
Ljung-Box Test R Q(20) 19.9813 0.4591
Ljung-Box Test R^2 Q(10) 13.91629 0.1768442
Ljung-Box Test R^2 Q(15) 14.979930.4528642
Ljung-Box Test R^2Q(20) 15.20514 0.7645518
LM Arch Test R TR^2 15.30931 0.2249553

Table i-4: Garch (1, 1) normalized distribution

Error Analysis:	Estimate	Std. Error	t value	Pr(>|t|)
Omega	1.671e-04	9.585e-05	1.743	0.0813
Alpha1	5.649e-01	3.745e-01	1.508	0.1315
Beta	12.858e-01	1.772e-01	1.613	0.1067

The condition of the coefficients is obeyed as shown below.

The sum of α+β<1 as shown by the values of alpha and beta,
0.5649+0.2858 = 0.8507 < 1, this is an indicator that volatility is persistence. The values of α and β are at 1% significance level.

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Table i-5: Garch (1, 1), Student T test.

Estimate	Std. Error	t value	Pr(>|t|)
Omega2.865e-04	2.377e-04	1.205	0.228078
alpha11.000e+00	7.147e-01	1.399	0.161750
Beta1 1.000e-08	1.342e-01	0.000	1.000000
Shape 2.817e+00	8.255e-01	3.413	0.000642 ***

The sum of $\alpha + \beta < 1$ with student-t distribution in GARCH(1,1),values are shown as:

$0.00000001 + 0.002817 < 1$

Table i-6: Garch (11) GED

Estimate	Std. Error	t value	Pr(>|t|)
Omega 2.446e-04	1.283e-04	1.907	0.0565
Alpha1 8.098e-01	6.061e-01	1.336	0.1816
Beta1 1.000e-08	1.525e-01	0.000	1.0000
Shape 1.000e+00	2.307e-01	4.334	1.47e-05

Sum of $\alpha+\beta<1$ as indicated by the values below,

$0.80978050+0.00000001<1$ Volatility is persistence in all the three distributions. The more accurate method of choosing the distribution is summarized in next section.

Selection Criterion For NORM, STD T And GED

We can use the AIC value to the one with the lowest value from the summarizedtable below.

Table i-7: AIC table for the three Distributions:

MODEL	DISTRIBUTION	SELECTION CRITERION	VALUES
GARCH11	NORM	AIC	-4.646305
GARCH11	STD t	AIC	-4.851885
GARCH11	GED	AIC	-4.852741

We choose on the model distribution which has the least AIC which Garch (1,1) is GED, in this case is GED Distributions has the lowest value.

Finally the distribution has been chosen which can be used in the GARCH (1,1) to forecast variance of the interest rates and Treasury bill rates discussed in the next page.

Forecasting Plots

Essentially any model chosen should be able to predict the possible future occurrences under given conditions such as Market, Economic conditions, etc. In the model, forecast for the next 12 months is projected as shown below.Figure below shows forecast interest rates from 55^{th} month to 67^{th} month.It shows the rates are farely constant. This is because of political stability ,agricultural produceis sufficient and there prices are stable.The prices of petroleum prices are farely constant.The interest rates are now controled the by the act of parliament.

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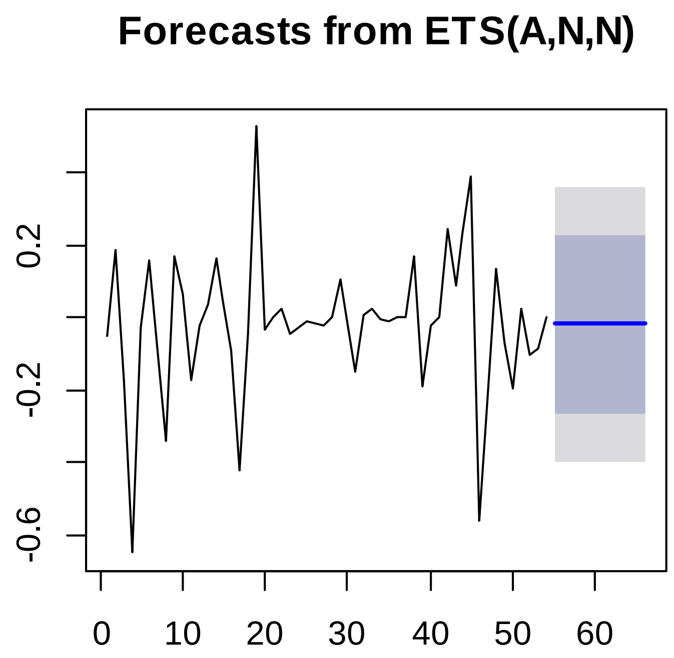

Figure i-5: Forecast plot for next 12 Months

From figure 10, it is clear that the interest rates will remain constant in the next 10 months shown at 5% prediction blue/dark shaded area, reason being measures taken by the government to reduce interest rates, availability of agricultural products, prices of petroleum products remaining constant .For the next 12 months predictions at 80% interval are shown in the shaded blue area, and at 95% prediction interval as grey shaded area.

Multifactor Model

Using the GARCH(1, 1) model, the standard deviation estimates for the variables are obtained. The standard deviation of interest rate was 0.603457. The treasury rate has a standard deviation of 0.58028. These values are represented by b_i's in the multifactor model. The base value a_i was set to be 300 billion shillings. Combining these values we get the multifactor model as:

$$P_i = 300,000,000,000 + 0.603457 I_1 + 0.580287 I_2 \quad (27)$$

Where P_i is the pension plan value at time i, I_1 is the interest rates, I_2 is the treasury rates.Using the equation of the multifactor model and substituting the standard deviations of the variables together with their forecasted values, we obtain the projected pension plan values for the next 10 years.
The projected 10 year pension plan values.

Table i-8: Forecast for the next 12 months.

Year	Pension plan Value

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1	2.567327×10^15
2	2.709412×10^15
3	2.851079×10^15
4	2.992966×10^15
5	3.134863×10^15
6	3.276769×10^15
7	3.418687×10^15
8	3.560617×10^15
9	3.702563×10^15
10	3.844382×10^15

The model above was used to forecast the growth of pension fund as the interest and Treasury bill rates change with time. Other factors affecting the fund such corruption that is becoming a virtue to some people was assumed constant, The projection is if 300billion is invested in the beginning of year 1 then after10 years fund would grow up to 3.844382×10^^{15}

From the model, it can be seen that when all other values remain constant at time zero, the pension plan performance would be 300 billion. This value was taken to be the base value in which the pension plan was assumed to have started with.

From the model, it can be seen that when all other values remain constant at time zero, the pension plan performance would be 300 billion. This value was taken to be the base value in which the pension plan was assumed to have started with. According to these values of the variables, there is evidence that the interest rate affected the pension plan performance more than Treasury bill rates.

Interpretation of Findings

From the above Multifactor Model, the study found out that macroeconomic variables and interest rates influence the pension plan performance more than the treasury rates. The study established that the coefficient for the interest rate is high meaning that the interest rate significantly influence the pension plan performance in Kenya. These finding contradicts the findings of Najarzadeh et al (2009) who found out that that the interest rates have a negative impact on the pension performance in long run and have a positive impact in the short term.

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CONCLUSION

The volatility of interest rates and Treasury rates was modeled using ARCH and GARCH(1, 1) and tested QQ-plots,it was seen clearly from results that volatility of interest rate varies with time. There were large variations followed by large variations of variance and also small changes followed by other small changes. The Treasury rates displayed same results. The GARCH(1, 1) was used with GED distribution to forecast the volatility for the next 12 months. Result showed volatility changes are small due to the fact that interest rates and Treasury rates are expected to remain fairly constant.

The Multifactor Model was used to assess the effects of volatility of interest and Treasury rates on pension fund. The study established that the coefficient for interest rate is high than that of Treasury rates meaning that interest rates significantly influence the pension fund performance in Kenya.

Limitations of the study.

The Treasury bill rates are given on weekly basis, the data has high frequency if analyzed on daily basis than monthly averages. I used monthly averages so that same model is applied to interest rates which were only available as monthly weighted averages.

There are many other factors affecting pension fund apart from interest rates and Treasury rates and are not quantified in the model therefore required more time to investigate

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